

Add. Ms. b. 186

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- ? 11 The function  $\vartheta(x)$ . Theorem IV.  $\vartheta(x) = O(x)$
- 13 Theorem V. Theorem VI. Proof
- ? 15 Introduction of  $\psi(x)$ . Ch. VII, Ch. VIII, Ch. IX
- 17 Ch. IX, Ch. X, Ch. XI, Ch. XII
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- ? 19 The Riemann  $\zeta$ -function,  $\zeta(s)$ . Euler's product for  $\zeta(s)$
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- 63 Proof of the Primzahlssatz

? p. 66 Proof that  $\zeta(s)$  is regular for  $\sigma > 0, s \neq 1$ . Lemma (1).  
Proof.

68 Lemma (2)

69 Lemma (3)

? 71 Riemann's Proof of the Functional Equation for  $\zeta(s)$

75 "Ueber die Anzahl der Primzahlen unter einer  
gegebenen Grösse" (Remarks)

99 Über Riemanns Nachlass<sup>[sic]</sup> zur analytischen Zahlentheorie  
C.L. Siegel in Frankfurt a. M. (Received 3.11.1931)  
(Quellen und Studien zur Geschichte der Math.  
(B) 2, 1933, 45-80.) [Synopsis here]

105 §1. Calculation of a definite integral

109 §2. The semiconvergent series for the  $\zeta$ -function

? 117 §3. The integral representation of  $\zeta(s)$

123 §4. Importance of the two Riemann formulae  
for the theory of the  $\zeta$ -function

137 Table of the Primes from 1 to 10,000

loose Primes from 10000000 to 10001000

Primes from 20000000 to 20001000

Primes from 20003000 to 20004000

Primes from 50000000 to 50001000

annotations in another hand: pp. [31v], [32v]-[46v], [74v],  
(? 75-[84v]-[94v]-135)